

# Function Transformations (1.12) Refresh

1.  $f(x)$  is a function with domain  $[-2, 4]$  and range  $[1, 3]$ . Find the domain and range of each of the following:

a.  $3f(x)$

c.  $f(2x)$

b.  $f(x - 1)$

d.  $f(-x)$

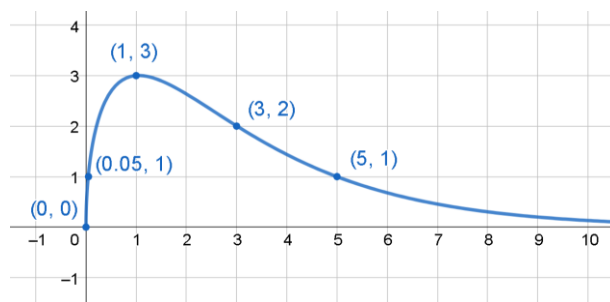
2. The table below gives values for the function  $f$  at selected values of  $x$ . Let the graph of  $g(x)$  be the image of the graph of  $f(x)$  after the following transformations done in the order listed: vertical dilation by a factor of 2, reflection over the  $y$ -axis, horizontal translation by  $-3$  units.  $g(x)$  may be written as  $g(x) = af(b(x + c))$  where  $a$ ,  $b$ , and  $c$  are constants.

|        |    |     |    |   |   |    |   |
|--------|----|-----|----|---|---|----|---|
| $x$    | -3 | -2  | -1 | 0 | 1 | 2  | 3 |
| $f(x)$ | 4  | -10 | -7 | 2 | 1 | -3 | 0 |

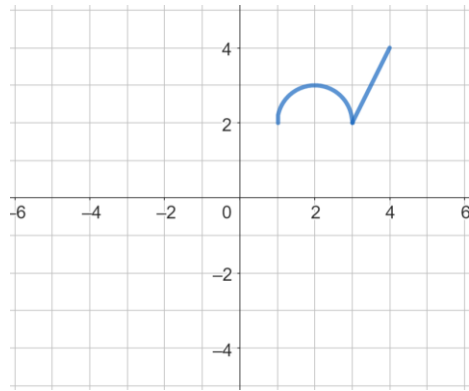
- a. Find the values of  $a$ ,  $b$ , and  $c$  to write  $g(x)$  in terms of  $f(x)$ .
- b. Find  $g(-2)$ .
3. The functions  $f$  and  $g$  are defined for all real numbers such that  $g(x) = -f(3(x - 1))$ . Which of the following sequences of transformations maps the graph of  $f$  to the graph of  $g$  in the same  $xy$ -plane?
- (A) A horizontal dilation by a factor of 3, followed by a reflection over the  $x$ -axis, followed by a horizontal translation by 1 unit
- (B) A horizontal translation by 1 unit, followed by a reflection over the  $x$ -axis, followed by a horizontal dilation by a factor of 3
- (C) A reflection over the  $x$ -axis, followed by a horizontal dilation by a factor of  $\frac{1}{3}$ , followed by a horizontal translation by 1 unit
- (D) A reflection over the  $x$ -axis, followed by a horizontal translation by 1 unit, followed by a horizontal dilation by a factor of  $\frac{1}{3}$
4. Consider two functions  $f(x)$  and  $g(x)$ . The graph of  $g(x)$  is the image of the graph of  $f(x)$  after the following transformations in the following order: a vertical translation by 3 units, a reflection over the  $x$ -axis, a horizontal dilation by a factor of 2. Write an expression for  $g(x)$  in terms of  $f(x)$ .
5. The point  $(-2, 10)$  is on the graph of  $y = f(x)$ . Find the corresponding point on the graph of  $y = g(x)$  if  $g(x) = 2f(x + 5)$ .
6. The point  $(-2, 10)$  is on the graph of  $y = f(x)$ . Find the corresponding point on the graph of  $y = g(x)$  if  $g(x) = f(2x) + 5$ .

7. The point  $(-2, 10)$  is on the graph of  $y = f(x)$ . Find the corresponding point on the graph of  $y = g(x)$  is  $g(x) = f(-x - 3)$ .
8. The point  $(-2, 10)$  is on the graph of  $y = f(x)$ . Find the corresponding point on the graph of  $y = g(x)$  if  $g(x) = f(-(x - 3))$ .

Consider the function  $g(x)$  graphed below. The function is defined over  $[0, \infty)$ , has horizontal asymptote  $y = 0$ , has a relative maximum value of  $f(1) = 3$ , and has its only point of inflection at  $(3, 2)$ . Use this function to answer questions #9-15



9. Over what interval does  $3g(-x)$  increase?
10. Over what interval is  $g(2(x - 1))$  concave down?
11. Over what interval does  $-g(x + 1)$  decrease?
12. At what  $x$ -value does  $g(-x) + 4$  switch between increasing and decreasing?
13. At what  $x$ -value does the rate of change of  $-g\left(\frac{1}{2}x\right)$  switch from increasing to decreasing?
14. Let  $h(x) = 2g(x - 1) - 3$ . Find the domain and range of  $h(x)$ .
15. Let  $h(x) = 2g(x - 1) - 3$ . Find  $\lim_{x \rightarrow \infty} h(x)$ .



Above is the graph of  $y = f(x)$ . Match each of the numbered functions below with the letter of the corresponding graph.

16.  $-f(2x)$

17.  $f(-x - 1) - 4$

18.  $-f(-x) + 4$

19.  $f\left(\frac{1}{2}x + 2\right) - 2$

20.  $f(-2x)$

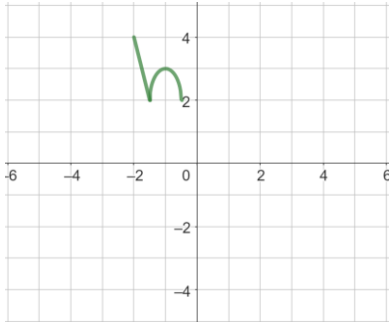
21.  $f\left(\frac{1}{2}(x + 2)\right) - 2$

22.  $f(x + 4) - 2$

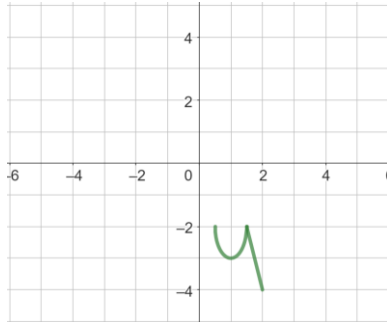
23.  $f(-(x - 1)) - 4$

24.  $2f(x + 1) - 4$

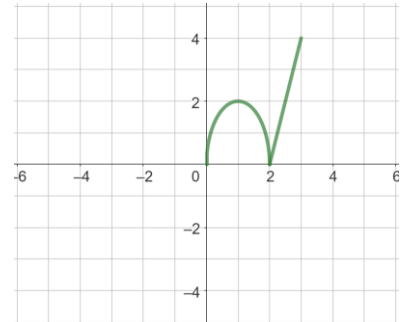
A



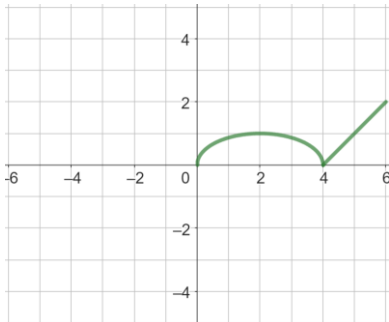
B



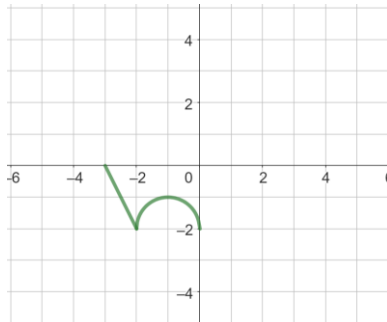
C



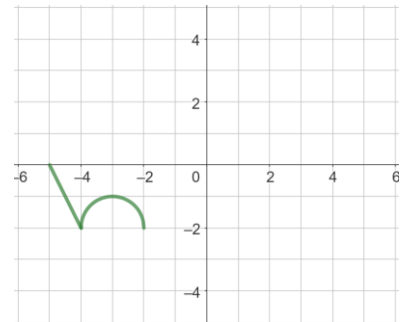
D



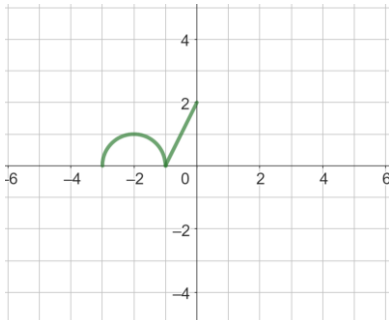
E



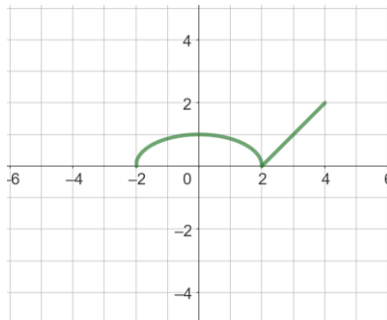
F



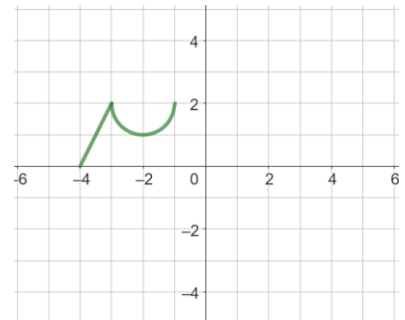
G



H



I



## Function Transformations Refresh HW Answers

1.
  - a. Domain:  $[-2,4]$     Range:  $[3,9)$
  - b. Domain:  $[-1,5]$     Range:  $[1,3)$
  - c. Domain:  $[-1,2]$     Range:  $[1,3)$
  - d. Domain:  $[-4,2]$     Range:  $[1,3)$
2.
  - a.  $g(x) = 2f(-(x + 3))$
  - b.  $g(-2) = 2f(-1) = -14$ .
3. C
4.  $g(x) = -f\left(\frac{1}{2}x\right) - 3$
5.  $(-7,20)$
6.  $(-1,15)$
7.  $(-1,10)$
8.  $(5,10)$
9.  $(-\infty, -1]$  (note: using the open interval  $(-\infty, -1)$  is fine as well)
10.  $\left(1, \frac{5}{2}\right)$
11.  $(-1,0)$
12.  $x = -1$
13.  $x = 6$
14. Domain:  $[1, \infty)$   
Range:  $[-3,3]$
15.  $-3$
16. B
17. F
18. I
19. H
20. A
21. D
22. G
23. E
24. C